IMAGE SOURCE SEPARATION USING COLOR CHANNEL DEPENDENCIES

8th International Conference on Independent Component Analysis and Signal Separation

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ICA 2009. March 15-18, 2009, Paraty, Brazil

PROBLEM STATEMENT

Unmixing of superposed images based on Bayesian formalism which uses prior information on color channels

SOURCE SEPARATION

The separation of images from their mixtures given a number of observations can be interpreted as an inverse problem.

$$\mathbf{Y} = \mathbf{A}\mathbf{S} + \mathbf{N}$$

- $\mathbf{A}$: Mixing matrix
- $\mathbf{S}$: Unknown source image (or images)
- $\mathbf{Y}$: Observation vector (or obsevation matrix)
- $\mathbf{N}$: Observation noise

Color Image Linear Mixing Model

Observed color image

$$\begin{bmatrix} y_r(n) \\ y_g(n) \\ y_b(n) \end{bmatrix} = \begin{bmatrix} a_{r,r} & 0 & 0 & a_{r,m} \\ 0 & a_{g,g} & 0 & a_{g,m} \\ 0 & 0 & a_{b,b} & a_{b,m} \end{bmatrix} \begin{bmatrix} s_r(n) \\ s_g(n) \\ s_b(n) \\ s_m(n) \end{bmatrix} + V$$

Color image sources

Achromatic source

Mixing matrix

Color pixels are contaminated with additive zero-mean Gaussian noise $\Sigma = \mathrm{diag}\{\sigma_r^2, \sigma_g^2, \sigma_b^2\}$

Problem Definition in Bayesian Framework

$$\underbrace{p(\mathbf{s}_{r,g,b,m}, \mathbf{A} \mid \mathbf{y}_{r,g,b})}_{\text{Joint posterior}} \propto \underbrace{p(\mathbf{y}_{r,g,b} \mid \mathbf{s}_{r,g,b,m}, \mathbf{A})}_{\text{Likelihood}} \underbrace{p(\mathbf{s}_{r,g,b,m}, \mathbf{A})}_{\text{Joint prior}}$$

Independency

$$p(\mathbf{s}_{r,g,b,m}, \mathbf{A} \mid \mathbf{y}_{r,g,b}) \propto p(\mathbf{y}_{r,g,b} \mid \mathbf{s}_{r,g,b,m}, \mathbf{A}) \, p(\mathbf{s}_{r,g,b}) \, p(\mathbf{s}_m) \, p(\mathbf{A})$$

Color image: Interchromatic dependence is taken into account.

Achromatic image

Posteriors

Under the assumptions of
- Mixing matrix and sources are independent
- Prior of the mixing matrix assumed to be uniformly distributed
- Achromatic image independent of color image components
- Color image components are mutually dependent

The intractable posterior can be simplified as:

$$p(\mathbf{s}_{r,g,b} \mid \mathbf{y}_{r,g,b}, \mathbf{A}, \mathbf{s}_m) \propto p(\mathbf{y}_{r,g,b} \mid \mathbf{s}_{r,g,b,m}, \mathbf{A}) \, p(\mathbf{s}_{r,g,b})$$

$$p(\mathbf{s}_m \mid \mathbf{y}_{r,g,b}, \mathbf{A}, \mathbf{s}_{r,g,b}) \propto p(\mathbf{y}_{r,g,b} \mid \mathbf{s}_{r,g,b,m}, \mathbf{A}) \, p(\mathbf{s}_m)$$

$$p(\mathbf{A} \mid \mathbf{y}_{r,g,b}, \mathbf{s}_{r,g,b,m}) \propto p(\mathbf{y}_{r,g,b} \mid \mathbf{s}_{r,g,b,m}, \mathbf{A})$$

Likelihood

$$p(y_{r,g,b} \mid s_{r,g,b,m}, \mathbf{A}) = \prod_{k \in \{r,g,b\}} \mathrm{N}\left(y_k \mid \bar{y}_k, \sigma_k^2\right)$$

Gaussian Mean Variance

$$\bar{y}_k = \sum_{l \in \{r,g,b,m\}}^{L} a_{k,l} s_l$$

Posterior of Mixing matrix

$$p(a_{k,l} \mid y_{r,g,b}, s_{r,g,b,m}, \mathbf{A}_{-a_{k,l}}) \propto N(a_{k,l} \mid \mu_{k,l}, \gamma_{k,l}) \underbrace{\left[u(a_{k,l}) - u(a_{k,l} - A_{\max})\right]}_{\text{To satisfy the positivity of mixing matrix}}$$

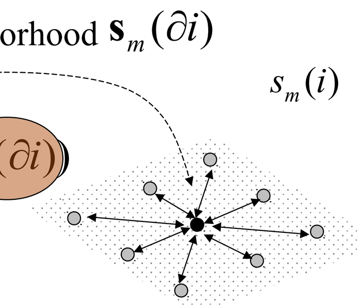
$$\mu_{k,l} = \frac{1}{\mathbf{s}_l^T \mathbf{s}_l} \mathbf{s}_l^T \left(\mathbf{y}_k - \sum_{i \in \{r,g,b\}, i \neq l}^{L} a_{k,i} \mathbf{s}_i \right) \qquad \gamma_{k,l} = \frac{\sigma_k^2}{\mathbf{s}_l^T \mathbf{s}_l}$$

Source Models

Achromatic Source Model

Posterior density of a single pixel i given its neighborhood $\mathbf{s}_m(\partial i)$

$$p(s_m(i) \mid \mathbf{y}_{1:K}, \mathbf{s}_{m\setminus(i)}) \propto p(y_{1:K}(i) \mid s_m(i)) \, p(s_m(i) \mid \mathbf{s}_m(\partial i))$$



Context in the MRF model

Prior density of a single pixel

$$p(s_m(m) \mid \mathbf{s}_m(\partial i)) = \frac{1}{Z(\beta_m)} e^{-\frac{1}{2}\sum_{j \in \partial i} \beta_m \rho_m (s_m(i) - s_m(j))}$$

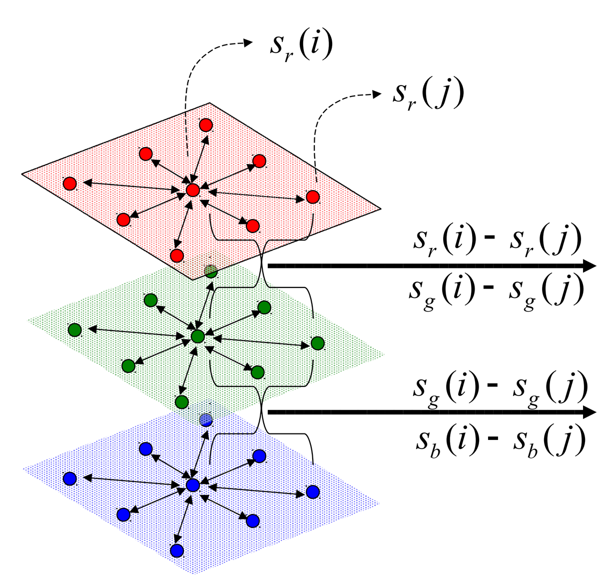
$$\rho_m(s_m(i) - s_m(j)) = \ln \underbrace{\left\{ 1 + \frac{(s_m(i) - s_m(j))^2}{\delta_m} \right\}}_{\text{Cauchy distribution}}$$

Partition function Clique potential

Color Source Model

$$\rho_c(\mathbf{s}_c(i) - \mathbf{s}_c(j)) = \frac{5}{2} \ln \left\{ 1 + (\mathbf{s}_c(i) - \mathbf{s}_c(j))^T \Delta^{-1} (\mathbf{s}_c(i) - \mathbf{s}_c(j)) \right\}$$

$$\mathbf{s}_c(i) = \left[s_r(i) \quad s_g(i) \quad s_b(i) \right]^T$$



Pixel differences of red and green channels are dependent.

Pixel differences of green and blue channels are dependent.

The cross-correlation matrix of pixel differences. We assume the red and green channels are independent, because they are far from each others in the spectrum.

$$\Delta = \begin{bmatrix} \delta_{r,r} & \delta_{r,g} & 0 \\ \delta_{r,g} & \delta_{g,g} & \delta_{g,b} \\ 0 & \delta_{g,b} & \delta_{b,b} \end{bmatrix}$$

Algorithm

- Optimization methods for the MAP estimation do not guarantee global solution because clique potentials are not convex.
- We resort to MCMC methods, i.e., Gibbs sampling.

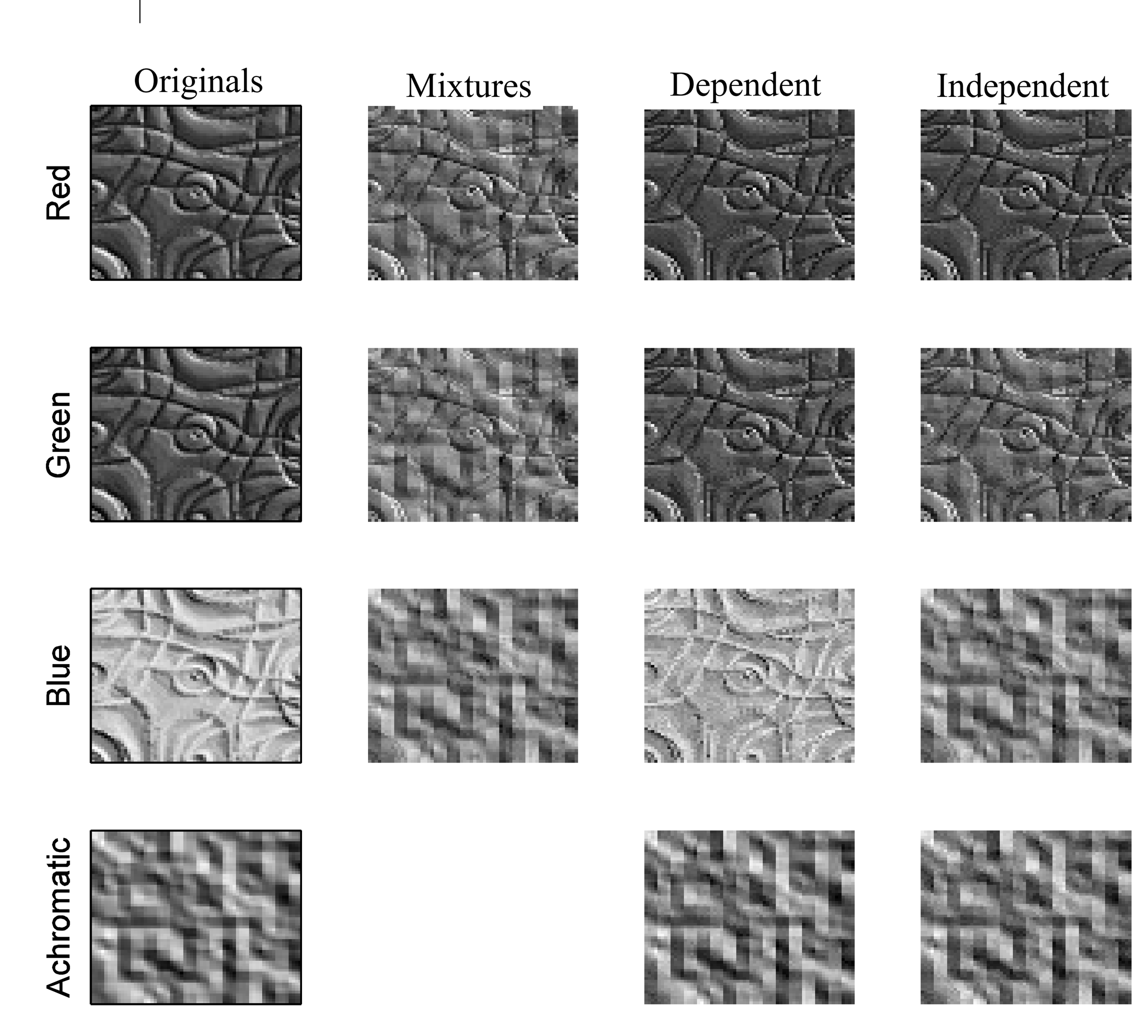
Gibbs sampling algorithm

> for all source image, $l = r, g, b, m$
>
> for all pixels, $n = 1 : N$
> Using Metropolis method
> $s_l^{t+1}(n) \longleftarrow \mathrm{sample}_{s_l(n)} \{ p(s_l(n) \mid \mathbf{s}_{l\setminus(n)}^t, \mathbf{s}_{\{r,g,b,m\}\setminus l}^t, \mathbf{y}_{r,g,b}, \mathbf{A}^t) \}$
>
> for all elements of mixing matrix, $(k, l) = (1, 1) : (K, L)$
>
> $a_{k,l}^{t+1} \longleftarrow \mathrm{sample}_{a_{k,l}} \{ p(a_{k,l} \mid \mathbf{A}_{-(k,l)}^t, \mathbf{y}_{r,g,b}, \mathbf{s}_{r,g,b,m}^t) \}$

SIMULATION RESULTS

Synthetic Mixture Case

- First column: Source images
- Second colum: Observed mixtures
- Third column: Separated sources using dependence model
- Fourth column: Separated sources without dependence model



- The mixing matrix is chosen as

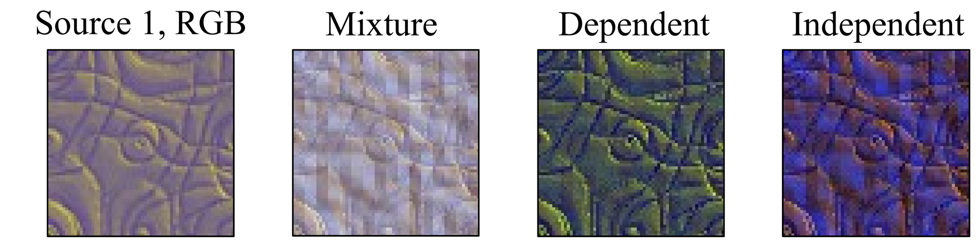
$$\mathbf{A} = \begin{bmatrix} 1.0 & 0 & 0 & 0.4 \\ 0 & 0.7 & 0 & 0.6 \\ 0 & 0 & 0.5 & 0.8 \end{bmatrix}$$

- The parameters of MRF model and noise variances are tuned by user.

Peak Signal-to-Interference Ratio (PSIR) results in dB:
- Average gain when dependent model used: 5.24 dB
- Peak gain when dependent model used: 10.64 dB
- The interference in the red channel is not strong, so the performances are comparable.

	red	green	blue	achromatic
Dependence assumption	48.98	37.69	36.39	38.55
Independence assumption	48.30	31.83	25.75	34.76

True color images: (a) original color texture, (b) true color mixture (observation), (c) and (d) estimated color texture under dependence and independence assumption, respectively



Real Mixture Case

For a realistic application, we used a color image taken with digital camera and corrupted by a reflection. The scene is organized such that a toy is standing behind a transparent CD box and a reflection occurs on the box surface. The reflection is assumed as an achromatic source while the scene behind is a color image source.

Removal of reflection image mixed to a color image. (a) Observed actual mixed image, (b) Color image with mixed reflection removed, (c) Estimated achromatic reflection image.

Estimated mixing matrix:

Red	Green	Blue	Achromatic
1.0283	0.0572	-0.2653	0.1463
0.3154	0.4069	-0.1128	0.3420
0.0222	-0.0217	0.3467	0.6297

CONCLUSIONS

- Statistical models in the form of color dependence prior information can significantly improve the performance of blind source separation algorithms.
- A case study: When not taking into account the dependence model, the algorithm has failed to separate the blue component in a synthetic mixture.
- The proposed algorithm can find application in color document analysis, restoration of ancient documents and polarized image applications.

Future work:

- Automatically estimate the MRF model parameters and noise variances in a Bayesian framework.
- Extend the model to include local dependencies in an image, a case in point being the non-stationary mixture case in the toy example with reflection.